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CONTRIBUTIONS FROM THE JEFFERSON PHYSICAL
LABORATORY, HARVARD UNIVERSITY.

ON THE ELECTROMAGNETIC AND THE THERMO-
MAGNETIC TRANSVERSE AND LONGITUDINAL
EFFECTS IN SOFT IRON.

BY EDWIN H. HALL AND L. L. CAMPBELL.

Presented February 8, 1911. Received February 8, 1911.

For many years past the electrical and thermal properties of a certain kind of wrought iron have been, from time to time, under investigation in this laboratory, and a number of papers¹ giving the methods and results of this study have already been published. The present paper has to do with the so-called Hall effect and the allied effects in the same kind of iron. These are so numerous and so unfamiliar to ordinary observation that it seems well to give an enumeration and brief description of them, somewhat like that given by H. Zahn² in his comprehensive paper on such phenomena.

In each of Figures 1-4 the rectangle, with very short side projections, or arms, represents pretty accurately, halfscale, the broadside of one of our "plates" of iron, which were each about 5 cm. long between heavy terminal blocks of copper (see Figure 7), and about 2 cm. wide. The circle indicates the size of the flat face of each pole of the electromagnet, the arrow showing the direction of the magnetizing current.

DESCRIPTION OF THE TRANSVERSE EFFECTS.

Hall Effect. — Figure 1, in which EE is the longitudinal electrical current, and $E'E'$ is the transverse electrical current due to the action of the magnet, shows the Hall effect, positive, according to the ordinary convention as to the sign, in this case, the equipotential line which would naturally extend straight across between the arms being rotated

¹ These Proceedings, **41**, May, 1905; **42**, March, 1907.

² Über galvanometrische und thermomagnetische Effekte, *Jahr. d. Rad. u. Elek.*, **5**, 166-218 (1908).

by the magnetic action through a small angle in the direction of flow of the Amperian magnetizing current.

The letter commonly used for the Hall effect coefficient is R ; we shall use also ${}_eT_e$ in the same sense, T indicating *transverse*, the initial e indicating that the *longitudinal* flow is *electrical*, the final e indicating that the *transverse* flow is *electrical*. If we let

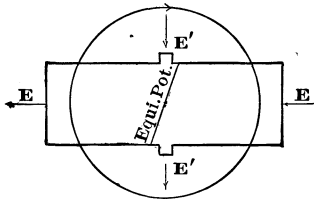


FIGURE 1.

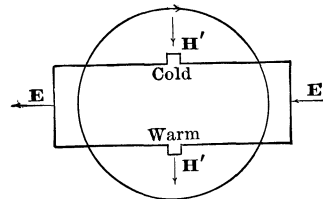


FIGURE 2.

- C = the longitudinal current, in absolute units,
 H = " intensity of the field " " "
 $\Delta P'$ = " diff. of pot. set up between the arms,
 w = " width of the plate in cm.,
 t = " thickness of the plate in cm.,

we have by definition

$${}_eT_e \text{ (or } R) = \frac{\Delta P'}{w} \div \frac{C}{wt} H = \frac{t \Delta P'}{CH}. \quad (1)$$

Ettingshausen Effect. — In Figure 2, with the longitudinal electrical current EE and the magnetic field just as in Figure 1, we take note of a transverse difference of temperature, $\Delta\theta'$, established by magnetic action. This is the Ettingshausen effect. $\Delta\theta'$, as here represented, is in such a direction that a transverse current of heat would flow in the same direction as the transverse electrical current of Figure 1. In this case the Ettingshausen effect, like the Hall effect of Figure 1, is said to be positive.

Zahn uses P for the Ettingshausen coefficient. We shall use also, in the same sense, ${}_eT_h$, the T , as before, standing for *transverse*, the e for *electrical*, longitudinal, the h for *thermal*, transverse. We have by definition

$${}_eT_h \text{ (or } P) = \frac{\Delta\theta'}{w} \div \frac{C}{wt} H = \frac{t \Delta\theta'}{CH}. \quad (2)$$

Nernst Effect. — In Figure 3, with the magnetic field as in Figures 1–2, we have a longitudinal heat-current, HH , and we take note of a transverse difference of potential, $\Delta P'$, established by magnetic action. This is the Nernst effect. This $\Delta P'$, as here represented, is in the same direction as the $\Delta P'$ of Figure 1, in which the longitudinal current was electrical. In this case, we shall call the Nernst effect positive; Zahn would call it negative. Zahn uses Q for the Nernst

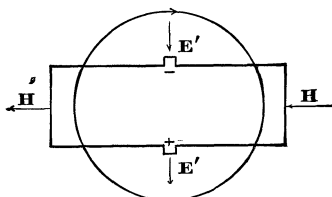


FIGURE 3.

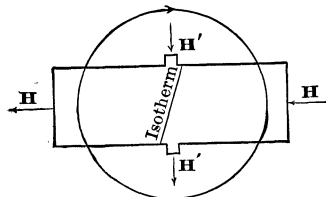


FIGURE 4.

coefficient with his convention as to the sign. We shall use ${}_hT_e$ as equivalent to his $-Q$, the prefix h indicating a *thermal* longitudinal flow, the T_e indicating a transverse electrical action. We have by definition

$${}_hT_e \text{ (or } -Q) = \frac{\Delta P'}{w} \div \frac{d\theta}{dl} H, \quad (3)$$

where $\frac{d\theta}{dl}$ = the temperature-gradient along the plate.

Leduc Effect. — In Figure 4, with the magnetic field and the longitudinal heat-current as in Figure 3, we take note of the transverse difference of temperature, $\Delta\theta'$, established by magnetic action. This is the Leduc effect. This $\Delta\theta'$, as here represented, is in the same direction as the $\Delta\theta'$ of Figure 2, in which the longitudinal current was electrical. In this case, the Leduc effect is called positive. Zahn uses S for the Leduc coefficient. We shall use also, in the same sense, ${}_hT_h$. We have by definition

$${}_hT_h \text{ (or } S) = \frac{\Delta\theta'}{w} \div \frac{d\theta}{dl} H, \quad (4)$$

The longitudinal effects are defined later, in connection with an account of our work on them.

DESCRIPTION OF APPARATUS.

Each of the iron plates mentioned above was brazed at each end into a thick copper block, as in Figures 7 and 8. After being so fixed each plate was measured in thickness at nine points, the distribution of which is well shown by Figure 5, the distance between (1) and (2), (2) and (3), (4) and (5), etc., being approximately 1.5 cm.

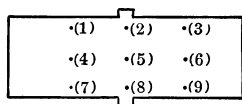


FIGURE 5.

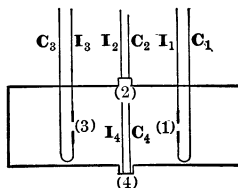


FIGURE 6.

Plate 1.—The measurements of Plate 1 were

Length between coppers 5.1 cm.,
 Width “ “ 1.98 cm.,
 Arms about 0.12 cm. long and about 0.10 cm. wide,

	cm.	Pt.	cm.	Pt.	cm.	Pt.
Thickness	0.1490	at 1	0.1460	at 2	0.1505	at 3
	0.1490	“ 4	0.1480	“ 5	0.1495	“ 6
	0.1475	“ 7	0.1450	“ 8	0.1500	“ 9
Means	0.1485		0.1463		0.1500	

As the part of the plate which is particularly important in the transverse effects, the effects that interest us most, is the narrow transverse strip including the arms, the thickness of this part is to be especially considered. This, according to our measurements, is about 0.1463 cm.; but as the method of measurement, by means of vernier calipers with faces of considerable width, must tend to give a maximum value, we take off about 1 per cent from the quantity just given and call the mean thickness of the median transverse strip 0.145 cm.

To Plate 1, eight wires were attached (Figure 6) by means of copper electrolytically deposited. Four of these, C_1 , C_2 , C_3 , C_4 , were of copper about 0.017 cm. in diameter, the others, I_1 , I_2 , I_3 , I_4 , were of annealed iron wire about 0.020 cm. in diameter. This iron wire was drawn from a portion of the same bar that furnished the plates 1 and 2. These fine iron wires were joined to somewhat heavier iron wires of the same source 15 cm. or more above the iron plate, and the heavier wires, perhaps a

meter long, were soldered to copper wires, the junctions being kept in a water bath. The distance between junctions (1) and (3) could not be very accurately determined. It was measured and taken as 2.78 cm. The wires C_1 and C_3 ran down from the points of attachment, passed under the lower edge of the plate, and came up across it on the other face. The distance between C_2 and I_2 and between C_4 and I_4 was perhaps 0.05 cm., each wire being attached to the outer end of the arm. On the face shown in Figure 6, a flat sheet of asbestos about 0.15 cm. thick, arranged so as not to disturb the wires, was cemented by means of asphaltum or melted shellac (the latter was used with Plate 2), the whole being subjected to a temperature high enough to soften the cement and allowed to cool with sufficient pressure on the asbestos sheet to fix it flat in place.

The width of Plate 1 is the direction of the fibre, or grain, of the original iron bar.

Plate 2. — The measurements of Plate 2 were

Length between coppers 5.15 cm.,
 Width 2.01 “,
 Arms about 0.10 cm. wide and 0.10 cm. long,

	cm.	Pt.	cm.	Pt.	cm.	Pt.
Thickness	0.134	at 1	0.135	at 2	0.138	at 3
(Figure 5)	0.138	“ 4	0.142	“ 5	0.145	“ 6
	0.140	“ 7	0.143	“ 8	0.146	“ 9
Means	0.137		0.140		0.143	

The face of Plate 2 was smoother at the last than the face of Plate 1, and we have taken as the mean thickness of the arm strip of (2) the mean of the measurements as here recorded, 0.140 cm. The thickness is several per cent greater on one edge of the plate than on the other, but it does not appear that any serious harm can come from this inequality.

The iron wires of Plate 1 (see Figure 6) were with Plate 2 replaced by constantan (“advance”) wires, K_1 , K_2 , etc., about 0.020 cm. in diameter, which had been annealed by heating to bright incandescence by means of an electric current. The distance from junction (1) to junction (3) was now made about 2.62 cm., in order to have them a little nearer the centre of the magnetic field than the corresponding junctions of Plate 1 had been. Wires C_1 and C_2 did not here pass under the lower edge of the plate, but followed the course shown in Figure 6, strips of mica fastened by shellac, melted on, being used for insulation. The wires themselves were bedded in shellac in their course across the plate. The plate was finally backed, on the side

shown, with a flat sheet of asbestos about 0.15 cm. thick, according to the description already given.

The length of Plate 2 is the direction of the fibre of the original bar.

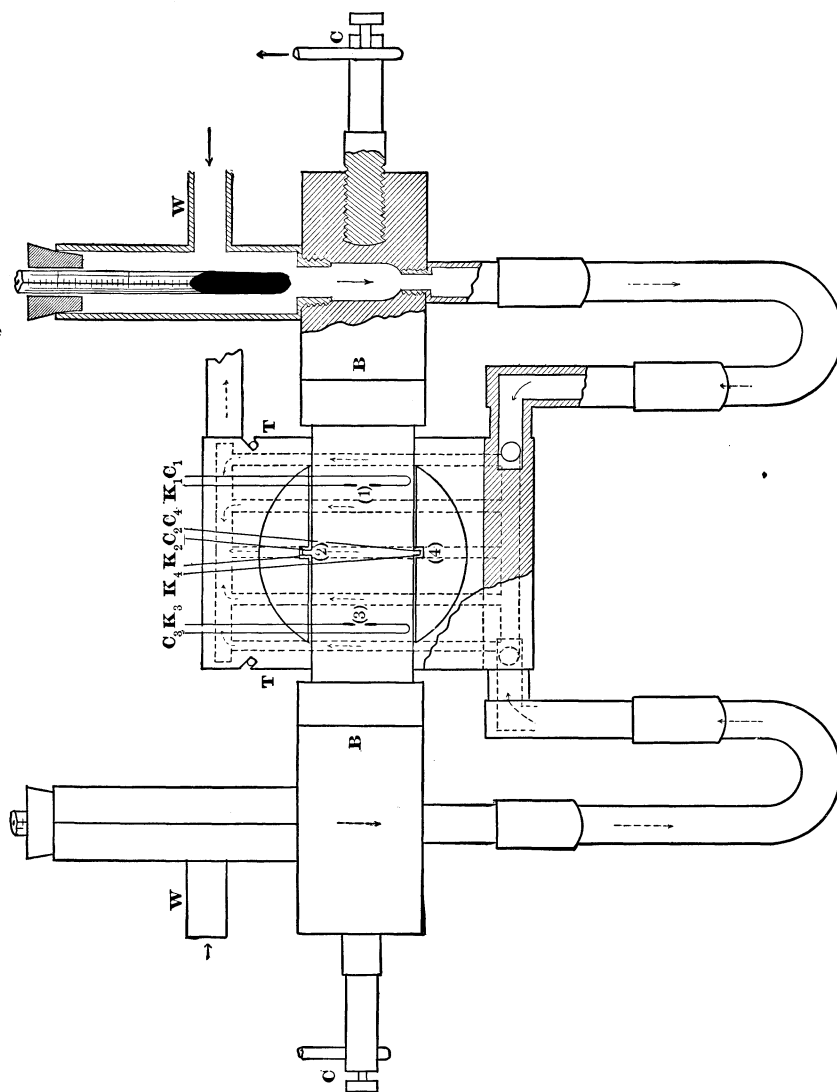


FIGURE 7.

MOUNTING THE PLATES IN
THE FIELD.

Figures 7 and 8 show the method of mounting the plates in the magnetic field for the electromagnetic experiments. *TT* is a trough of soft iron consisting of a rectangular one-piece³ block about 4.5 cm. long, 6.5 cm. tall and 2.0 cm. wide, with a channel 0.5 cm. wide and 5.5 cm. deep cut through it from end to end, leaving an uncut part 1 cm. thick below. Through holes drilled in this uncut portion two streams of water, entering at the ends and meeting at the middle, reach ten vertical passages, five in each wall of the trough. The passages in each wall lead into a horizontal passage at the top, and the two streams of water issuing from the apparatus are reunited into one beyond the range of our figures.

The object of this water circulation is to control the temperature of the trough, and so that of our iron plate, which lies in the channel of the

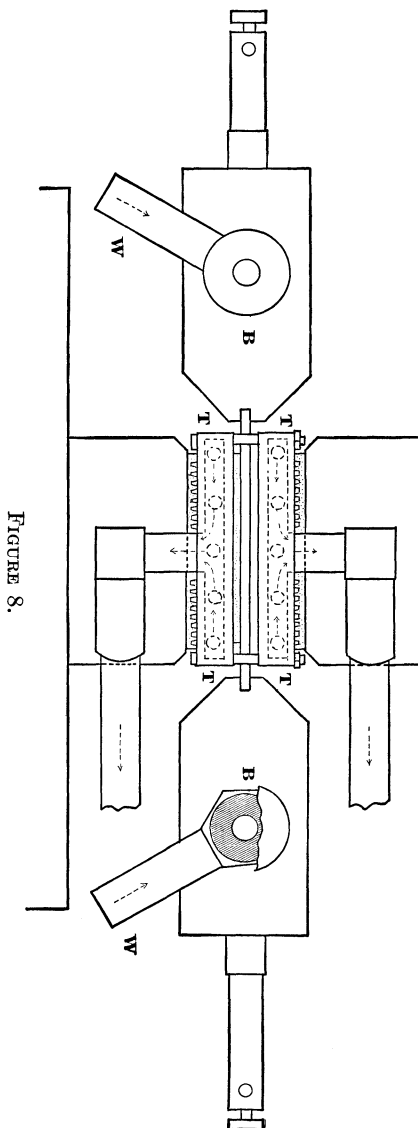


FIGURE 8.

³ The trough first used was not from one piece, but was a composite affair soldered together. It leaked more or less, and after much trial the new trough was substituted for it March 26, 1910. Nearly all our observations on the transverse and longitudinal effects made before that date we regard as tentative only and reject from our final results.

trough, during the electromagnetic observations; for in these it is desirable to have all parts at the same temperature, except as differences may be set up by the magnetic action that we are studying.

The asbestos pad, indicated by a dotted strip in Figure 8, at the back of the plate bears against one face of the channel in the trough. Between the plate and the other face of this channel is a space about 0.2 cm. wide, into which passes the single-turn "test-coil," of 1 cm. radius, by means of which the strength of the magnetic field is measured. The test-coil is in circuit with a ballistic galvanometer and the throw produced when it is suddenly withdrawn from the field is easily and accurately measured. The semicircular pieces above and below the plate are of soft iron of nearly the same thickness as the plate, their object being to insure uniformity of magnetic field over the whole width of the iron under observation. Care is taken to have the plate a little nearer to one face of the channel than to the other, so that it may not be pulled forward on the test-coil side by magnetic action. At each end of the trough near the top is a slot which holds a small brass bar, having a head at one end and a nut at the other and carrying a stud 0.5 cm. long on its middle. This device prevents the walls of the trough from bending toward or from each other during the magnetic action.

During thermomagnetic observations also, with a longitudinal gradient of temperature, the plate was at first kept in the iron trough, without, of course, circulation of the water therein. It appeared, however, after a time, that the nearness of the walls of the trough affected the longitudinal temperature-gradient, and after May 21, 1910, the trough was discarded during thermomagnetic measurements. At this time disks of cork (see Figure 8) about 0.2 cm. thick were cemented to the flat faces of the pole pieces of the magnet, which faces are 4.0 cm. in diameter, and the exposed surface of each disk was cross-cut with a file or saw, in order to reduce the naturally low heat-carrying power of the cork. The asbestos pad at the back of the iron plate was then placed against the cork disk on one pole of the magnet, and the other pole was brought somewhat toward the plate, but not near enough to pull the plate away from the other pole. Most of the thermomagnetic data which we use were obtained after this arrangement had been adopted.

The cork disks remained in use during all the subsequent measurements, even when the iron trough was put back for repetition of electromagnetic observations. Previous to their introduction a facing of asbestos had been used between the pole pieces and the sides of the trough.

The tubes W and W show where the two streams of water enter the apparatus. The course of these streams is first downward past the thermometer-bulbs, which give the temperature readings called T_n and T_s , through the massive copper blocks B and B . Then, if electromagnetic experiments are in order, the course followed is that shown by the arrows in Figure 7; but if a temperature-gradient for thermomagnetic work is to be established and maintained, the two streams, one warm and the other cold, pass off at once from B and B to the sink.

The longitudinal electric current, when one is used, enters by one of the heavy wires C , C , and passes out by the other.

Plate 1 was studied at intervals from June 5, 1909, to June 29, 1910. All the observations we have to publish on Plate 2 were made during December, 1910.

CALIBRATION OF THE THERMO-ELECTRIC COUPLES.

The "thermo-electric height" between copper and our iron was determined some years ago for various temperatures ranging from 27° to 71° , and from this set of data a curve was plotted which gives the thermo-electric force of a copper-iron couple within these limits. As our iron wires were drawn from a piece of the same iron bar that furnished the iron used in determining these thermo-electric heights, and as they were annealed after drawing, it was assumed that the curve just described would serve for couples made by joining these wires with copper; that is, we assumed the iron wires to be thermo-electrically so like the plate to which we attached them, that we should be justified in ignoring any thermo-electric forces which might arise from differences between the plate and this wire.

But to our surprise and disappointment we found that the thermo-electric force of the couple I_1 -plate- I_s was about one tenth-part as that of the couple C_1 -plate- C_s . This discovery, which was not made at the beginning of our work, taken with the fact that the thermo-electric quality of iron is considerably affected by magnetization, led to the rejection of iron wires and the use of constantan wires in their place, when we came to the preparation of Plate 2.

Experiments to determine whether constantan wire would be appreciably effected in its thermo-electric quality by magnetism were conducted as follows: A piece of annealed constantan wire was fastened by means of shellac, applied melted, within a narrow glass tube across, or through, which ran side-tubes carrying streams of water, one at a temperature near 16° degrees, the other near 76° .

The distance between the streams was about 6 cm., so that the gradient of temperature in the wire between must have been about 10° per cm. The ends of the wire, which was about 130 cm. long, were soldered to copper wires, and the junctions were placed in a suitable water bath, the copper wires leading to the galvanometer. The part of the iron wire lying between the streams of hot and cold water was placed across the space between the magnetic poles in a field the strength of which, for a distance of perhaps 4 cm., was not far from 8000.

Galvanometer readings taken with "field off" and "field on" showed that the change of thermo-electric quality produced in the unequally heated constantan wire by magnetic action, if any such change occurred, was probably too small to yield a change of 3×10^{-8} volt in the thermo-electric force of the circuit.

Four couples of copper and annealed constantan wires were tested for calibration purposes. The temperatures used were those of the tap-water (about 12.5°), boiling ether (about 35.5°), boiling alcohol (about 80°), and boiling water. The method used was substantially that which has been described in previous papers.⁴ None of the junctions came into direct contact with any one of the liquids or its vapor, each being protected by an arrangement like that shown in Figure 10. The thermometers used had been studied with considerable care in previous work.

The four couples agreed so closely that for no one of the temperature intervals used in the calibration did the result from any one couple differ so much as one per cent from the mean result given by all. This mean follows :

Temperature interval.	Mean e. m. f. per degree.
11.6° – 34.5°	3911 absolute
34.6° – 79.1°	4145 "
79.3° – 99.9°	4527 "

The thermo-electric current runs from constantan to copper at the hot junction ; that is, at ordinary temperatures the copper line on the thermo-electric diagram lies between the iron line and the constantan line.

Plotting a curve through the three points given by the data shown above, we get from it the following table for copper-constantan :—

⁴ These Proceedings, 41, p. 42.

Temperature.	Thermo-electric height (microvolts per degree).	Temperature.	Thermo-electric height (microvolts per degree).
0° C.	38.04	60° C.	41.72
10°	38.46	70°	42.71
20°	38.94	80°	43.91
30°	39.50	90°	45.33
40°	40.14	100°	47.10
50°	40.88		

MEASUREMENT OF THE TRANSVERSE EFFECTS.

In giving the results of our observations we shall report on the various transverse effects in the order in which we have defined them. Relations, theoretical or empirical, between some of them will be discussed later, with especial reference to the formulas of Moreau and of Voigt.

Hall Effect. — With Plate 1 we found

1910.	ϵT_e .	θ .	C_p .	H .
Mar. 29	+ 714 $\times 10^{-5}$	12.7°	2.9	6500
" 30	1024 "	53.0°	"	"

θ here is the temperature of the water flowing through the copper blocks at the end of the plate and the iron trough in which the plate lay. C_p is approximately the strength of the current through the plate, expressed in absolute units; H is approximately the strength of the magnetic field at the middle part of the plate, expressed in absolute units.

The temperature-coefficient of ϵT_e , as derived from the data here given, is

$$(1024 - 714) \div 714 (53 - 12.7) = 0.0108.$$

Thermo-electric action, due to the Ettingshausen transverse temperature-difference acting in the all-iron thermo-electric couple I_2 -plate- I_4 of Figure 6, is negligible in its influence on the Hall effect as here observed.

With Plate 2, using the circuit C_2 -plate- C_4 , we found

1910.	ϵT_e .	θ .	C_p .	H .
Dec. 3	+ 807 $\times 10^{-5}$	13.1°	2.94	5150
" 22	781 "	12.7°	2.97	5460
	794 "	12.9°	2.96	5300
" 10	1486 $\times 10^{-5}$ *	84.5°	2.99	5040
	(1464) "			

After the observations of December 10, on the Hall effect, it was found that the plate had been several millimeters too high in the magnetic field during the observations of December 3 and 10, so that the strength of the field to which it had been exposed was somewhat uncertain. Proper adjustment was made December 22, and later in the same day the Hall effect was measured again at low temperature. The coefficient found December 22 was about 3.3 per cent lower than that of December 3, though the temperature on the later date was only 0.4° lower, which would account for scarcely 0.5 per cent difference. For the value of ${}_eT_e$ at the mean low temperature, 12.9° , we have taken the mean, 794×10^{-5} of the values found on December 3 and December 22. For the value of ${}_eT_e$ at the high temperature, 84.5° , we have taken the value found December 10 reduced by 1.5 per cent, thus getting 1464×10^{-5} .

For the temperature-coefficient of ${}_eT_e$ in Plate 2, we get

$$(1464 - 794) \div 794 (84.5 - 12.9) = 0.0118.^5$$

Ettingshausen Effect. — With Plate 1 we undertook to measure this effect by using the east and the west coils of our galvanometer differentially, connecting the I_2I_4 circuit with the west coil and the C_2C_4 circuit with the east coil and taking care to have the total resistance of one circuit equal to that of the other. This arrangement we called (I_2-I_4) *versus* (C_2-C_4). Lest there might be error from a possible accidental advantage of one coil of the galvanometer over the other in action upon the needle, a series of observations made with this arrangement was always coupled with a (C_2-C_4) *vs.* (I_2-I_4) series, in which the C_2C_4 circuit was connected with the west coil and the I_2I_4 circuit with the east coil.

The theory of this test was that the Hall effects in the two circuits would just neutralize each other, leaving the Ettingshausen effect in the C_2C_4 circuit to give an account of itself in the net result. This experimental method was not so good as that used in the study of the

⁵ Dr. Alpheus W. Smith gave in the Physical Review for Jan., 1910, the results of his study of the Hall effect in various metals through a large range of temperature. Taking data from the diagrams of his paper, we find as the temperature-coefficient of the Hall effect

0.0088	between 22° and 100° in "Kahlbaum iron,"
0.0140	" 23° " " " electrolytic iron.

The mean of these values, 0.0114, agrees very well with the values found by us in Plates 1 and 2.

Ettingshausen effect in Plate 2, and the results here attained are not entitled to any great confidence. They are

1910.	ϵT_h .	θ .	C_p .	H .
Mar. 29	$- 375 \times 10^{-10}$	12.7	2.86	6500
" 30	$- 1120$ "	53.1	2.85	"

It seems inadvisable to calculate a temperature-coefficient of ϵT_h from these data.

With Plate 2 we put the circuit C_4K_4 into connection with the west coil of the galvanometer and the circuit C_2K_2 into connection with the east coil, as in Figure 9.⁶ We shall call this arrangement C_4K_4 *vs.* C_2K_2 , or (4) *vs.* (2). Series with this arrangement were coupled with series having a (2) *vs.* (4) arrangement, the galvanometer connections being exchanged in the shift from one arrangement to the other. The results are

1910.	ϵT_h .	θ .	C_p .	H .
Dec. 9	$- 500 \times 10^{-10}$	12.2°	2.94	5000
	[492]			
" 10	621 "	83.9°	30.0	"
	[612]			

The numbers given in brackets, which are the ones to be finally taken, are reached by deducting 1.5 per cent from the numbers immediately above, the deduction being made for reasons given in connection with ϵT_e .

From these data we get the following temperature-coefficient of ϵT_h in Plate 2 :

$$(612 - 492) \div 492 (83.9 - 12.2) = 0.0034.$$

The transverse differences of temperature (between the arms) actually produced by electromagnetic action in the cases here described in Plate 2 were about 0.0053° at the low temperature and 0.0067° at the high temperature. Whether these temperature-differences are set up instantaneously by the magnetic action or whether they grow for a time after the full magnetic field-strength is on, is a question we cannot at present answer with full confidence. The galvanometer was usually read about 45 to 60 seconds after the magnet-current, C_m , was put on. We have not observed any tendency toward galvanometer

⁶ Proportions are not observed in this figure, nor is the device for keeping the two wires separate, except at their soldered junction, within the glass tubes in *V* shown. For this detail see Figure 10.

drift of such a character as to indicate that we should have got larger Ettingshausen effects by waiting longer for these effects to be established.

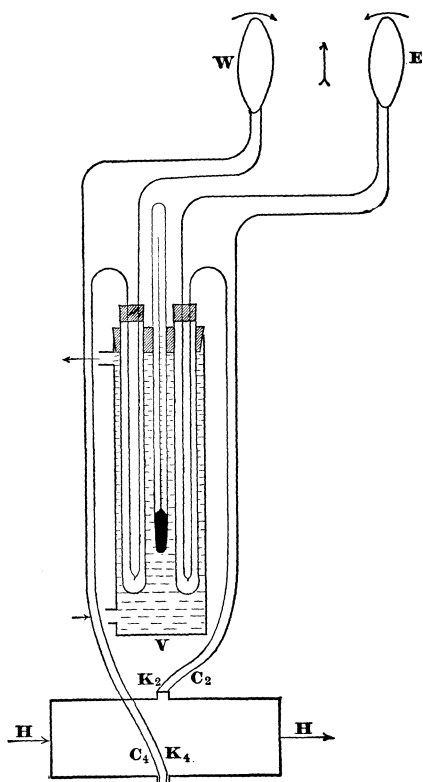


FIGURE 9.

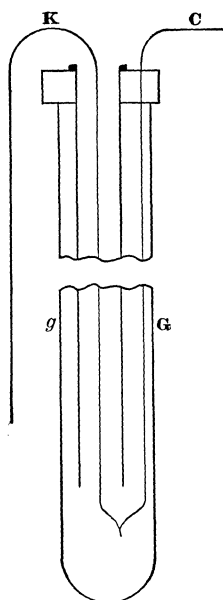


FIGURE 10.

The Nernst Effect.— With Plate 1 we at first used the iron trough already described, during the thermomagnetic tests. We now reject all the Nernst-effect observations made with this trough between the poles. The remaining results are

1910.	${}_h T_e$.	Arm θ .	$\frac{d\theta}{dl}$.	H .
June 4	-944×10^{-6}	45.7°	11.3°	8400
" 10	928	44.8°	11.3°	8700
Means	-936	45.3°	11.3°	8550

The “arm θ ,” that is, the temperature of the median cross-line of the plate, was found by connecting either the C_2I_2 circuit or the C_2I_4 circuit with the galvanometer and then bringing the outer junction of the thermo-electric couple to such a temperature as to make the current in the circuit zero.

The temperature-gradient $\frac{d\theta}{dl}$ along the plate was found by connecting the thermo-electric circuit C_1 -plate- C_3 with the galvanometer.

The transverse potential difference, the Nernst effect, was found by means of the circuit I_2 -plate- I_4 , the assumption being made that the thermo-electric difference between the iron plate and the iron wires could safely be neglected, so that no account need be taken here of the fact that a transverse temperature-difference, the Leduc effect, is set up simultaneously with the Nernst effect. It now seems very doubtful whether this assumption was justified; but as we have good reason for supposing that the error thus introduced into the value of ${}_hT_e$ was not more than 10 per cent, and as we are not sure of the sign of this error, we leave ${}_hT_e$ for Plate 1 as we have found it.

With Plate 2 the conditions were better. We found arm θ and $d\theta/dl$ in essentially the same way as before. We found the transverse e. m. f., $\Delta E'$ let us say, produced in the circuit C_2 -plate- C_4 by magnetic action and deducted from this the amount, a large part of the whole, which from a study of the Leduc effect we found to be attributable to a thermo-electric force set up in the circuit C_2 -plate- C_4 by magnetic action. If we call this e. m. f. $\Delta_i E'$ and call the true Nernst transverse difference of potential $\Delta P'$, we have

$$\Delta P' = \Delta E' - \Delta_i E.$$

Thus at 31° we found $\Delta P' = 132.3 - 54.5 = 77.8$ absolute,

and at 60° “ “ $\Delta P' = 137.0 - 50.3 = 86.7$ “

The results are

1910.	${}_hT_e$	Arm θ .	$\frac{d\theta}{dl}$.	H.
Dec. 28	— 880×10^{-6}	30.1°	8.4 (N-S)	5550
“ 29	840 “	31.7°	8.3 (S-N)	“
Means	860 “	31.0°	8.4	5550
Dec. 30	940×10^{-6}	59.3°	7.7 (S-N)	5560
“ 30	1020 “	60.3°	7.7 (N-S)	5470
Means	— 980 “	60.0°	7.7	5500

Reference to formula (3) defining ${}_hT_e$ will show that it contains the factor $1 \div \left(w \times \frac{d\theta}{dl}\right)$, where w is the width. Now as the Nernst $\Delta P'$ is

found between two points separated by the width of the main plate plus the length of both arms, it might seem as if w must be taken as this whole distance. On the other hand, as the flow of heat in the arms follows a somewhat dubious course, one may well doubt whether the whole length of the arms should be included. Fortunately, the fact that the width w occurs in combination with the gradient $\frac{d\theta}{dl}$ relieves us from this difficulty. The longitudinal gradient which counts is the gradient on the narrow strip running across from arm to arm and out into the arms, and this gradient will bear the same ratio to the measured gradient $\frac{d\theta}{dl}$ that the width of the main part of the plate bears to the effective width at the arms. Accordingly, we can take w as the width of the plate without the arms and $\frac{d\theta}{dl}$ as the gradient which would exist if the arms were absent, that is, practically, the gradient as measured by means of the circuit C_1 -plate- C_3 . This we have done, here and in dealing with the Leduc effect.

The temperature-coefficient of ${}_hT_e$ we find to be

$$(980 - 860) \div 860 (60 - 31) = 0.0048.$$

The Leduc Effect. — All of the Leduc observations on Plate 1 were made with the iron trough in use. For this reason, and the further reason that the dubious thermo-electric quality of our iron wire was involved in them, these observations are entitled to far less weight than those made with Plate 2.

The method used with both plates was essentially that indicated in Figure 9, the (2) circuit and the (4) circuit being used differentially, one connected with the west coil of the galvanometer, the other connected with the east coil.

With Plate 1 we found

1910.	${}_hT_e$	Arm θ .	$\frac{d\theta}{dl}$.	H.
Mar. 31	$+453 \times 10^{-9}$	43 (?)	11.0 (N-S)	6500
Apr. 2	410 "	44 (?)	11.3 "	"
Means	432 "	43.5(?)	11.2	6500
Apr. 1	421×10^{-9}	43 (?)	11.3 (S-N)	6500
" 2	438 "	44 (?)	11.3 "	"
" 9	420 "	45 (?)	11.3 "	"
Means	426 "	44 (?)	11.3	6500
Final means	$+429$ "	44 (?)	11.3	6500

With Plate 2 we found

1910.	${}_hT_h$.	Arm θ .	$\frac{d\theta}{di}$.	
Dec. 28	$+ 549 \times 10^{-9}$	29.8°	8.3 (N-S)	5600
“ 29	586 “	31.3°	8.6 (S-N)	5550
Means	$+ 568$ “	30.6°	8.5	5600
Dec. 30	$+ 614 \times 10^{-9}$	59.2°	7.9 (S-N)	5470
“ “	649 “	61.1°	7.7 (N-S)	5550
Means	$+ 632$ “	60.2°	7.8	5500

The temperature-coefficient of ${}_hT_h$ in Plate 2 is, according to these figures,

$$(632-568) \div 568 (60.2-30.6) = 0.0038.$$

The + sign of ${}_hT_h$ means, according to our definition, that the isothermal lines are by the magnetic field rotated in the same direction in which the magnetizing current flows.

The difference of temperature actually produced between the arms by this action was about 0.0054° .

The magnet current was usually put on about 45 seconds before the final galvanometer reading showing the Leduc effect was taken. Professor Campbell, who did most of the experimental work in this research, is under the impression that the Leduc effect did not attain its maximum value as soon as the full field-strength was reached, but that it continued, according to the testimony of the galvanometer, to grow for a time after that moment. We cannot speak positively, as yet, on this interesting particular.

RELATIONS OF ${}_eT_e$, ${}_eT_h$, ETC. : THE MOREAU FORMULA AND ITS SUGGESTIONS; THE VOIGT FORMULA.

Attempts have been made by various writers to correlate some or all of the various effects which are dealt with in this paper and to connect them with some of the more generally familiar electrical and thermal properties of metals. Naturally the electron theory has been used of late in such efforts; but we shall consider first a suggestion or proposition made by Moreau,⁷ about ten years since, in which no mention of the electron theory was made or intended. Indeed, Moreau says, “As, moreover, the variations of $K[{}_eT_h]$ are explained by those of $c[{}_eT_e]$, since the Hall effect is the primordial phenomenon, one is justified in supposing that the molecular state of the plate has a consider-

⁷ C. R., **130** (1900), pp. 122-124, 412-414, 562-565.

able influence on the value of c . This seems to me a new proof that the Hall phenomenon is due to a deformation of the plate under the influence of the magnetic field, and it is enough to explain the differences, often very great, observed between the values of c , for the same body, by different experimenters."

It is difficult to see how anyone who has tried the Hall effect with a variety of conditions as to the manner of supporting the metal under examination can attribute this effect to any ordinary distortion of the material. If distortion, or deformation, is the cause of the phenomenon in question, this distortion must be something as remote from ordinary bending and twisting as are the changes of molecular orientation which we imagine to accompany or constitute magnetization.

We might, therefore, pay little attention to the suggestion of Moreau, if he had not given and supported with some experimental evidence an exceedingly simple formula,

$${}_eT_e \div \rho = {}_hT_e \div s, \quad (5)$$

in which

${}_eT_e$ = the Hall coefficient,

${}_hT_e$ = " Nernst "

ρ = " specific electrical resistance,

s = " mechanical equivalent of the Thomson-effect coefficient, that is, the amount of heat energy (ergs) absorbed by the unit current (absolute) of electricity per second in going, according to the ordinary convention of current direction, through the metal from a point where the temperature is T degrees C. to a point where it is $(T + 1)$ degrees C.

This formula of Moreau, if it could be substantiated by further experimental evidence, would prove disturbing to the point of view from which the present definition of the Hall coefficient was framed, even if it did not lead to the conclusion that the Hall effect and its various allied effects are due to a deformation of some kind in the material showing these phenomena. For it is to be noted that in the definition of ${}_eT_e$ the relation of the transverse potential-gradient to longitudinal *current-strength*, not to the longitudinal potential-gradient, is expressed. The reasons for this are two: 1st, a notion held by the discoverer at the time of the discovery that the new effect was due to the action of magnetic force on a *current* of electricity, as such, not to the action of such force on a medium containing lines of electrical force,⁸

⁸ At the suggestion of Professor Rowland an attempt was made to rotate by magnetic force the equipotential lines of a piece of glass under electrical stress. The result was negative.—E. H. H.

as such; 2d, the fact that it was much easier to measure current-strength than potential-gradient, in the longitudinal direction, in metal sheets of uneven thickness and uncertain quality such as were often used in the early days of the Hall effect.

It is true that before long it began to be seen that metals of high resistance had, as a rule, high values of ϵT_e , and this fact raised the question whether high transverse potential-gradient might not go naturally with high longitudinal potential-gradient as such. But it was found that, in non-magnetic metals at least, rise of temperature, increasing the resistance and the longitudinal potential-gradient, did not as a rule, if ever, increase⁹ the Hall effect proportionally. Accordingly, there has seemed to be no sufficient reason for recasting the definition of ϵT_e by introducing the potential-gradient instead of the longitudinal current-strength.

But the formula of Moreau suggests, or perhaps is suggested, by such a change of view as this recasting of ϵT_e would imply. To prove this statement an argument leading to this formula will now be given.

We will treat the matter first in its qualitative aspect; and we will take the case of our soft iron, in order to be the more definite. Figure 1, which is correct for this iron, shows a +Hall effect, the equipotential lines being rotated in the direction of the Amperian current of the field. Such a rotation establishes a transverse potential-gradient, the lower edge of the plate attaining thus the higher potential.

Turning now to Figure 3 and remembering that in iron the Thomson-effect coefficient, s , is negative, so that heat is absorbed by an electrical current when it flows (in the ordinary conventional sense) from high temperature to low temperature in iron, we see that in the present case we must, in order that an electrical current may *not* flow along the plate, have the cold end of the plate at a higher potential than the hot end; that is, we have, under the conditions of Figure 3, a potential-gradient opposite in direction to the temperature-gradient and opposite to the potential-gradient along the plate in Figure 1. Accordingly, our magnetic field, rotating the equipotential lines of this *static* Thomson-effect potential-gradient, would make in our iron the upper edge of the plate electrically positive as compared with the lower edge, thus giving what we call a negative Nernst effect. The Nernst effect in our iron is negative, according to our convention as to signs, and the

⁹ "The temperature-coefficient of the Hall effect in gold, zinc, platinum, silver and aluminum has . . . been determined and it has been found that with the exception of aluminum there is in each of these cases a decrease in the effect as the temperature is raised from -190° to about 22° ." Dr. Alpheus W. Smith, *Physical Review*, Jan., 1910.

signs at least of the Moreau formula are correct in this case, ${}_eT_e$ and ρ being both + and ${}_hT_e$ and s both - .

The quantitative argument is very simple. Writing $\frac{dP}{dl}$ for the potential-gradient which maintains the current C through a plate of width w , thickness t , and specific resistance ρ , we have, using absolute units, $C = \frac{dP}{dl} \times \frac{\omega t}{\rho}$, and accordingly

$${}_eT_e = \frac{\Delta P'}{w} \div \frac{C}{\omega t} H = \left(\frac{\Delta P'}{w} \div \frac{dP}{dl} \right) \frac{\rho}{H}. \quad (6)$$

Taking, as before (equation (3)),

$${}_hT_e = \frac{\Delta P'}{w} \div \frac{d\theta}{dl} H,$$

and observing that the *static* Thomson-effect potential-gradient is

$${}_s \frac{d\theta}{dl} = \frac{dP}{dl},$$

we have
$${}_hT_e = \left(\frac{\Delta P'}{w} \div \frac{dP}{dl} \right) \frac{s}{H}. \quad (7)$$

If, now, we can assume that the equipotential lines of the electric current in the Hall effect are by magnetic action rotated just as far as the static thermo-electric equipotential lines of the heat current in the Nernst effect, we shall have the parenthesis in equation (6) equal to the parenthesis in (7), and then, dividing (6) by (7), we shall get

$${}_eT_e \div {}_hT_e = \rho \div s, \quad \text{or} \quad {}_eT_e \div \rho = {}_hT_e \div s, \quad (8)$$

which is the equation of Moreau.

Before proceeding further it will be well to assemble in one table the various coefficients under discussion, each evaluated, as accurately as our knowledge of the rates of change with temperature will enable us to evaluate it, for various temperatures. We have, the last line relating to Plate 1 and the other lines to Plate 2,

Temp.	${}_eT_e$	${}_eT_h$	${}_hT_h$	${}_hT_e$	ρ	s	S
20°	861×10^{-6}	-505×10^{-10}	545×10^{-9}	-815×10^{-6}	12550	-851	1429
40°	1048	538	588	897	13730	961	1370
60°	1235	572	630	980	14910	1079	1307
80°	1422	604	672	1062	16090	1203	1240
100°	1609	638	714	1144	17270	1333	1171
45°	963	-971(?)	430(?)	-935			

(I.)

Previous work¹⁰ has given us 11365 as the value of ρ at 0°C., with a temperature-coefficient 0.00519, and a Thomson-effect formula, running from 32° C. to 182° C., from which we get

$$s = -(107 T + 2 T^2) 419 \times 10^{-5}, \quad (9)$$

T being the absolute temperature.

S of the last column is the estimated "thermo-electric height" of our iron relative to lead. It will be discussed at considerable length in connection with the formula of Voigt.

Testing the Moreau formula with the data given, we find

	Temp.	$(eT_e + \rho) = A.$	${}_hT_e + s = B.$	$(A - B) \div A.$	
Plate 2	20° C.	$+ 686 \times 10^{-9}$	$+ 958 \times 10^{-9}$	$- 0.40 -$	(II.)
	40°	763	934	$- 0.22 +$	
	60°	828	908	$- 0.10 -$	
	80°	884	883	$- 0.00$	
	100°	932	858	$+ 0.08 -$	
Plate 1	45°	$+ 686$	$+ 944$	$- 0.38 -$	

Not all of the temperature-coefficients are sufficiently well known to justify us in pushing the comparison through a greater range of temperature, and in fact the values here set down for A and B are subject to an error of several per cent, probably. But when all reasonable allowance for error has been made, it seems unlikely that the Moreau formula can hold in this iron through any considerable range of temperature. The difference between the result from Plate 1 at 45° and that for Plate 2 at or near the same temperature is less significant as throwing a doubt upon the accuracy of our work than it would be if the two plates had been cut in the same direction from the original iron bar. Plate 2 is far more important, not merely because it has been studied under better conditions, but because the main current in it was in the direction of the fibres of the iron, the direction along which the measurements for ρ and for s have been made.

It appears that the Moreau formula holds in our iron for one temperature, which in Plate 2 is near 80° C. Below this temperature the static Thomson equipotential lines appear to be rotated through a somewhat greater angle, by a given strength of field, than the equipotential lines of an ordinary electric current; but above this temperature the opposite appears to hold.

Comparison of $eT_e \div \rho$ in Table II with ${}_hT_h$ in Table I shows an

¹⁰ These Proceedings, 42, March, 1907.

approach to equality, which may be interpreted as meaning that the equipotential lines of the longitudinal electric current in the Hall effect and the isothermal lines of the longitudinal heat-current in the Leduc effect are rotated in the same direction and to nearly the same extent, at certain temperatures. Detailed comparison gives

Temp.	$(eT_e \div \rho) = A$	$hTh = B$	$(A - B) \div A$	} (III.)
20°	$+ 686 \times 10^{-9}$	$+ 545 \times 10^{-9}$	$+ 0.21 -$	
40°	763 “	588 “	0.23 -	
60°	828 “	630 “	0.24 -	
80°	884 “	672 “	0.24 -	
100°	932 “	714 “	0.23 +	

It would seem from this showing that, within the range of temperature here considered, the isothermal lines are rotated in the Leduc effect about three-fourths as far as the equipotential lines are rotated in the Hall effect, the strength of magnetic field being the same on both cases.

Comparison of Tables II and III indicates that, within the range of temperature shown, the static Thomson equipotential lines are rotated farther in the Nernst effect than the isothermal lines are simultaneously turned in the Leduc effect, but that the excess noted diminishes rapidly with rise of temperature and seems likely to become zero or negative with further rise.

As we have in the Moreau formula a suggestion at least of some relation between eT_e and hT_e , we might expect to find evidence of a reciprocal relation between hT_h and eT_h ; that is, between the Leduc effect and the Ettingshausen effect. Reflection shows, however, that one link in the chain of reasoning, if it deserves such a name, is here missing. We go from the Hall effect to the Nernst effect by way of the Thomson effect, the known difference of electric potential which accompanies flow of heat or difference of temperature in iron under normal conditions. But no corresponding difference of temperature necessarily accompanying flow of electricity or difference of electric potential is known to us. On the other hand, if we could find by inspection of the various coefficients before us evidence of a relation between hT_h and eT_h , we might perhaps take this as a clue to what could be called the reciprocal of the Thomson effect, and could be described as a temperature-gradient due to potential-gradient. The possibility of such a discovery makes it desirable to examine closely all the relations which a study of our various tables of data suggest.

A look at the uppermost line of Table I shows that, for the temperature 20° C. at least, we have, very nearly,

$${}_eT_e \times {}_eT_h = {}_hT_h \times {}_hT_e, \text{ or } {}_eT_e \div {}_hT_e = {}_hT_h \div {}_eT_h. \quad (10)$$

If we test this formula through the various temperatures we find

Temp.	$({}_eT_e \div {}_hT_e) = A$	$({}_hT_h \div {}_eT_h) = B$	$(A - B) \div A$	
20°	— 10.56	— 10.79	— 0.02 +	} (IV.)
40°	11.68	10.93	+ 0.06 +	
60°	12.60	11.01	+ 0.13 —	
80°	13.39	11.11	+ 0.17 +	
100°	14.07	11.19	+ 0.20 +	

The approximate equality here indicated appears to be closer than that of the Moreau formula. But is it anything more than accident? Going back to the definitions of ${}_eT_e$, etc., we find, putting $\Delta P_1'$ for the transverse potential-difference of the Hall effect and $\Delta P_2'$ for the transverse potential-difference of the Nernst effect,

$${}_eT_e \div {}_hT_e = \left(\frac{\Delta P_1'}{w} \div \frac{C}{wt} \right) \div \left(\frac{\Delta P_2'}{w} \div \frac{d\theta}{dl} \right) = \frac{\Delta P_1'}{\Delta P_2'} \times \left(\frac{d\theta}{dl} \div \frac{C}{wt} \right). \quad (11)$$

The dimensions of this quotient A are those of a temperature-gradient divided by a current density.

We find also

$${}_hT_h \div {}_eT_h = \left(\frac{\Delta \theta'_2}{w} \div \frac{d\theta}{dl} \right) \div \left(\frac{\Delta \theta'_1}{w} \div \frac{C}{wt} \right) = \frac{\Delta \theta'_2}{\Delta \theta'_1} \times \left(\frac{C}{wt} \div \frac{d\theta}{dl} \right). \quad (12)$$

The dimensions of this quotient B are the reciprocal of those of the quotient A , and accordingly it appears probable that we have only an accidental coincidence in the approach to equality shown by A and B .

If we undertake to explain the Ettingshausen effect as due to the magnetic rotation of the isothermal lines of a longitudinal temperature-gradient set up by the electric current along the plate, we see that, as the coefficient ${}_eT_h$ is about 0.09 as large as ${}_hT_h$, we must assume our temperature-gradient in question to be about 0.09 of a degree per centimeter in our plate when the electric current density therein is 1. Such a condition, as a normal attendant of a current in iron, could not have escaped the attention of ordinary experience. If it were set up by the act of magnetization, it would be detected in the longitudinal tests presently to be described.

Is it, then, possible that the Ettingshausen transverse effect is due

to an actual displacement of the electric current in our plate, the flow, under the action of the magnet, being stronger along one edge than along the other edge of the plate? This seems very unlikely, especially in view of the fact that, in Plate 2 at least, the resistance appears to be slightly, very slightly, decreased by magnetization. Indeed, the whole amount of heat generated per minute per unit length of the plate by our current C_p was about 0.6 calorie, and the change in this amount due to the magnetization was probably not so much as 0.00002 calorie.

There is at least one other point of view which is worth trying. The transverse potential-gradient set up in the Hall effect is, apparently, like the Thomson potential-gradient, *static*, maintained without flow of electricity down the gradient, though there may be, if the circuit is properly closed, flow of electricity up this gradient. In the Thomson effect this static condition is attended by, and is apparently due to, a temperature-gradient of the opposite slope. Is it perhaps true that wherever such a static potential-gradient exists in a metal there is an accompanying temperature-gradient, and are the two gradients in iron always of opposite slope? The Ettingshausen temperature-slope is opposite in our iron to the potential slope of the Hall effect, and to this extent encourages our question; but when we come to the ratio of the two slopes and compare this with the ratio of the two slopes in the Thomson effect, we find a great difference. In the latter case the ratio is our s . The exhibit follows:

Temp.	$(\epsilon T_c + \epsilon T_h)$	s .	$(\epsilon T_c + \epsilon T_h) \div s$.	(V.)
20°	— 170500	— 851	200.4	
40°	194800	961	202.7	
60°	216000	1079	200.1	
80°	235000	1203	193.3	
100°	252200	1333	189.2	

We have at least the satisfaction of finding that the ratio of the two ratios, although each changes a good deal, nearly 50 per cent, remains pretty nearly constant,¹¹ especially at the lower temperatures, where the values of ϵT_h are probably more accurate than for the higher ones. Indeed, the whole change from 20° to 100° is less than 6 per cent. But the transverse temperature-gradient is only one two-hundredth part of what it should be to make the transverse ratio equal to s .

It would be difficult to go farther in this line of inquiry without

¹¹ It should be remembered, however, that the temperature-coefficient of ϵT_h cannot be regarded as accurately determined.

with the line for lead. The representative lines for copper and iron are here made straight, though in a later figure they are shown as curves. In accordance with the ordinary interpretation of a temperature-entropy diagram, unit quantity of electricity, ten coulombs, flowing (in the ordinary sense) from C_0 to C in copper or from I to I_0 in iron absorbs heat, and *vice versa*, the Thomson effect; flowing from C in copper to I in iron, it absorbs heat, and *vice versa*, the Peltier or Seebeck effect.

If now we denote by $-\Theta_c$ ¹⁶ the area $T_0TCC_0T_0$ and by $-\Theta_i$ the area $T_0TII_0T_0$, the total, or net, thermo-electric force, in a copper-iron couple having one junction at T_0 (absolute zero) and the other junction at T , is represented by the area

$$C_0CII_0C_0 = -\Theta_i + \Theta_c. \quad (14)$$

If we let α_c = the line T_0C_0 and $\alpha_i = T_0I_0$, we can write

$$-\Theta_c = \alpha_c T + \frac{1}{2} \beta_c T^2, \quad (15)$$

and

$$-\Theta_i = \alpha_i T + \frac{1}{2} \beta_i T^2, \quad (16)$$

where

$$\beta_c = (C'C \div T) \quad \text{and} \quad \beta_i = -(I'I \div T).$$

These equations (15) and (16) correspond to equation (55) of Voigt's paper.

It is evident from our diagram that, if we allow a very small increase of temperature, from T to $T + dT$, and if we let S_i represent the entropy of ten coulombs at I , *the conventional zero of entropy being so taken that $S_i = TI$* , we have

$$-\frac{d\Theta_i}{dT} dT = S_i dT, \quad \text{or} \quad -\frac{d\Theta_i}{dT} = S_i. \quad (17)$$

Now the Θ' of the Voigt formula (13) is $\frac{d\Theta_i}{dT}$, and is therefore equal to our $-S_i$. It is numerically equal to the "thermo-electric height" of iron relative to lead.¹⁷

The relation of Θ'_i to s_i , the mechanical equivalent of the Thomson-effect coefficient, is readily found. The mechanical equivalent of the Thomson-effect heat absorbed by ten coulombs in rising in iron from

¹⁶ We use the — sign here in order to conform to the use of Voigt.

¹⁷ At least it has been so taken by Moreau in his empirical test of the Voigt formula, without protest, so far as we know, from Voigt, though the latter does not, perhaps, say explicitly that his Θ'_m for any metal m has reference to lead.

T to $T + dT$ is $s_i dT$, or rather, since s_i is essentially —, the mechanical equivalent of the heat *given out* in this change is $-s_i dT$, and this is represented by the area between the two adjacent vertical lines of our diagram. Evidently this area is equal to the area between the two adjacent horizontal lines from I' to I , that is, to $(a_i + \Theta'_i) dT$, and so we get

$$-s_i = \Theta'_i + a_i, \quad \text{or} \quad \Theta'_i + s_i = -a_i. \quad (18)$$

That is, the sum of Voigt's Θ'_i and the s_i , which Moreau uses in place of Θ'_i , is a constant, represented in Figure 11 by the line $T_0 I_0$ taken as a negative quantity. As we have already identified Θ'_i numerically with the line TI , taken as a negative quantity, we see that s_i is represented by the line II' taken as a negative quantity.

It must be remembered that this simple relation between s_i and Θ'_i results from the assumption made, for the time, in drawing Figure 11, and made also by Voigt, that the iron line is straight. The actual relation between s_i and Θ'_i is, according to our experiments, more complex.

The argument for Voigt's proposition, as expressed in equation (13), we shall not undertake to give; and even when we try to put it to an empirical test by means of our data, we do so with a serious *a priori* doubt as to its value. What real significance can we attach to the formula of Voigt? His $-\Theta'$, the "thermo-electric height" of iron with respect to lead, cannot be taken as a *fundamental* datum for iron. We have, to be sure, identified $-\Theta'$ with our S ; but this S does not profess to be the absolute value of the thermo-electric entropy for iron. It is merely what we get for iron when we arbitrarily take the thermo-electric entropy of lead to be zero.

We have proceeded as follows: Assuming the "thermo-electric height" of "galvanoplastic" copper relative to lead to be 380×10^{-8} microvolts, or 380 absolute, at 20°C ., from the observations of Matthiessen, and taking the thermo-electric height of our iron with respect to modern commercial copper wire (assumed to be thermo-electrically like Matthiessen's copper) to be 1049 absolute at 20°C ., according to our own observations, we get 1429 absolute as the thermo-electric height of our iron with respect to lead at 20°C . But "thermo-electric heights" are merely isothermal entropy-differences, if we reckon entropy, as we do, in $\text{ergs} \div T$. Accordingly in Figure 12 we take the T -axis as the lead line and at 20°C . lay off from this axis a distance 380 to give us the 20° -point on our copper line, and a distance of 1429 to give us the 20° -point on the iron line. The other points of the iron line must be found by means of our formula (9) for the Thomson effect in iron.

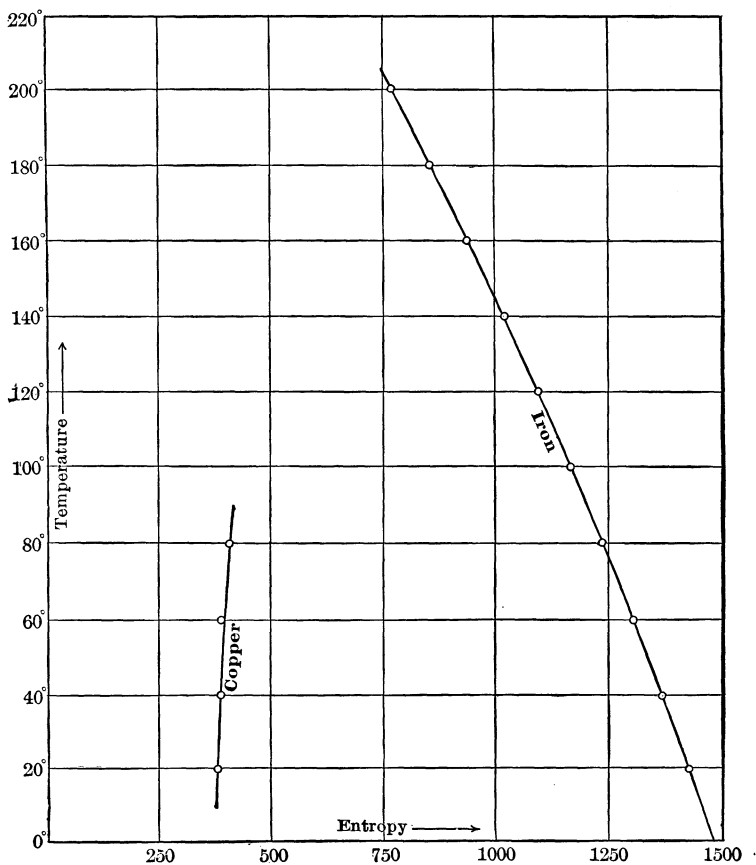


FIGURE 12.

Taking from Figure 11 the relation $TdS = sdT$, we get from (9)

$$dS = -419 \times 10^{-5} (107 + 2T) dT \dots \quad (19)$$

Integrated this gives

$$S = K - (107T + T^2) \times 419 \times 10^{-5} \dots \quad (20)$$

The value of S being taken, on grounds already shown, as 1429 at 20°C ., we find the value of K to be 1921. Accordingly the values of S have been calculated which are given in the following table :

Temp.	S (entropy).	Temp.	S .
0° C.	1486	120° C.	1098
20°	1429	140°	1021
40°	1370	160°	941
60°	1307	180°	858
100°	1171	200°	771

(VI.)

With these values of S the iron line of Figure 12 has been plotted. Various points of the copper line have been found by measuring to the left from the corresponding points of the iron line distances representing the thermo-electric heights of iron with respect to copper at the various temperatures. The iron line is decidedly concave toward the left, the copper line, according to our data, has a much smaller inclination from the vertical than the iron line, indicating a much smaller Thomson effect in copper than in iron, and is slightly concave toward the right.

If we now, taking our values of S for certain temperatures, e. g. 20°, 40°, 60°, 80°, 100°, test the Voigt formula for these temperatures, we find,

Temp.	$(-{}_eT_e S \div \rho) = A.$	${}_hT_e = B.$	$(A - B) \div A.$
20°	— 980×10^{-6}	— 815×10^{-6}	0.17 —
40°	1046 “	897 “	0.14 —
60°	1083 “	980 “	0.10 —
80°	1096 “	1062 “	0.03 +
100°	1091 “	1144 “	— 0.05 —

(VII.)

Comparison of this table with Table II. will show that the Voigt formula, with the value of $-S$ (or Θ') found as we have found it, agrees with our data considerably better than the Moreau formula does. Each formula is right in our case as to signs¹⁸ and each is satisfied by our data at some one temperature, which is near 80° C. for the Moreau formula and not very many degrees higher for the Voigt formula. Above the temperature of agreement ${}_hT_e$ is too small for the Moreau formula and too large for the Voigt formula. It is to be remembered that neither ${}_eT_e$ nor ${}_hT_e$ has been determined very accurately, nor is the temperature-coefficient of either precisely known. It seems to us, however, very unlikely that our errors of measurement are great enough to account for the disagreement between either formula and our data. It is probable that neither formula is correct.

¹⁸ There may be some doubt as to the signs. See the Postscript to this paper.

Of the two, the Moreau formula seems to us the more likely to be useful, as suggestive of true relations which it does not perfectly express. The value of s_0 , which we need to know in order to use this formula, we do know or can find. It is a definite property of iron and of iron alone. But the value of Voigt's Θ'_i , as we have shown, is merely a property of iron with relation to lead. Now we know that the lead line cannot be regarded as the real boundary of the thermo-electric diagram. The lines for certain metals lie beyond it, below it in the ordinary diagram, to the left of it in such a diagram as that of Figure 11. These metals have a negative thermo-electric height, a negative thermo-electric entropy, as regards lead. Reckoning thermo-electric heights from lead, taking lead as having zero thermo-electric entropy, is quite as arbitrary as taking the freezing point of water for the zero of temperature. We may do injustice here; it is possible that we have not seen the exact meaning of Voigt's Θ' .

However useful the formula of Moreau may be, his thesis that the Hall effect and all the transverse effects associated with it are due to some deformation of the metal plate in the magnetic field appears to be incompetent to explain the observed facts, especially the Ettingshausen effect, and, if taken alone, misleading rather than helpful. It is altogether probable that the electron theory must be used, if all the phenomena observed are to be accounted for.

THE LONGITUDINAL EFFECTS.

All of the transverse effects just considered change direction with change of direction of the magnetic field, the sign of the coefficient in each case remaining unchanged. The associated longitudinal effects, which are now to be described, remain, as the argument from symmetry would predict, unchanged in direction with change of direction of the field. One might, from this absence of dependence on the sign of the field magnetism, expect the longitudinal effects to be proportional to the square of the field-strength, and this may be true of some of these effects in some metals, but in general it appears not to be true. The fact seems to be, however, that the longitudinal effects are much less simple functions of field-strength than the transverse effects are and sometimes change direction with change of intensity of the field. Whether they are strictly or very nearly proportional to the longitudinal gradients of potential and temperature respectively, our own experiments do not enable us to say. We shall, however, assume this proportionality and shall accordingly define a certain coefficient for each of the two longitudinal effects which we have found in iron. For the sake of

completeness we shall describe two other longitudinal effects which have been observed, though we have not yet made sure of them in iron.¹⁹

No one of the four longitudinal effects has any assigned name. Each must be designated by a rather long descriptive title.

Longitudinal Electromagnetic Potential-Difference. — With the magnetic field and the EE current as in Figure 1, we may take note of a change of longitudinal potential-gradient, due to transverse magnetic action, without change of strength of the longitudinal current. Of this effect Zahn says, "As its magnitude is found to be proportional to the strength of the primary [longitudinal] current-strength, it may be taken as a change of resistance."

We shall call this effect positive when it produces an increase of resistance. We take as the coefficient

$${}_eL_e = \Delta R \div RH, \quad (21)$$

where R = the normal resistance of a certain length of the plate and ΔR = the increase of resistance of the same length of the plate.

In studying this effect in Plate 1 we connected points (1) and (3) of the plate (see Figure 6), by means of the iron wires, with our galvanometer, introducing into the galvanometer circuit e. m. f. from another circuit, so as to get an approximate balance of forces at the start, thus keeping the scale of the galvanometer within the field of view. Then we applied the magnetic field and observed the deflection produced, which was due to an increase or decrease in one of our counter-balancing e. m. f.'s, the other being not affected by the magnet. This method is troublesome, because it is difficult to keep the requisite degree of equilibrium through an extended series of observations.

This operation gave us for Plate 1 values of ${}_eL_e$ which are exhibited in the following table. The numbers given in the last three columns are approximate only.

1909.	${}_eL_e$.	Arm θ .	C_p .	H .
July 8	$+ 164 \times 10^{-10}$	$22^\circ 5$	2.96	10500
" 9	148 "	22.0	"	10900
Means	$+ 156 \times 10^{-10}$	22°	2.96	10700

With Plate 2 a different method was used. Copper wires connected with points (1) and (3) of the plate led to one coil of the galvanometer, while wires from two other points, A and B (see Figure 13), of the plate-circuit led to the other coil. By means of the shunts S_1 and S_2

¹⁹ But see p. 664, footnote.

an approximate balance of the galvanometer forces was obtained which remained fairly constant when the span of copper wire AB was kept in a bath of oil to prevent sudden changes of temperature. With this arrangement the following result was obtained with Plate 2 :

	$eLe.$	Arm $\theta.$	$C_p.$	$H.$
Dec. 24, 1910	-59×10^{-10}	$13^\circ 0$	3.0	5400

The value of R was about 113,000, absolute, or 0.000113 ohm, and that of ΔR about -3 , absolute. This is a small quantity, and it

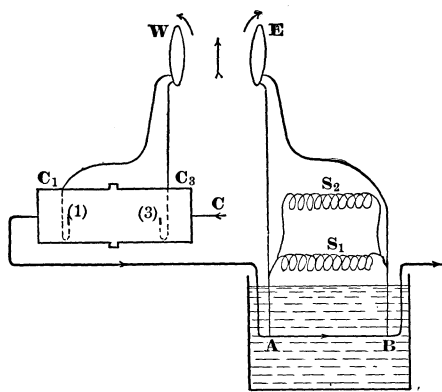


FIGURE 13.

may therefore be well to give a few details as to the observations. Five series, or "runs" were made, three with the current C_p flowing south through the plate, and two with this current flowing north. It is not safe to depend on observations made without such reversal of the main current; for there is always a possibility of some illegitimate action of the electromagnet circuit on the galvanometer which can be eliminated by combining two series of data, one obtained with the current

in the plate running north, the other with this current running south. The five series mentioned gave the following mean galvanometer deflections :

With C_p north.	With C_p south.
+ 0.33 cm.	+ 0.11 cm.
+ 0.24 "	- 0.21 "
	- 0.22 "
Means $\overline{+ 0.29}$ "	$\overline{- 0.11}$ "

Final mean = $\frac{1}{2} (0.29 + 0.11) = 0.20$ cm.

The first series with C_p south, which gives a mean deflection discordant in sign with the others, was the shortest series and internally the worst of the five.

One other possibility of error must be considered here. The copper wires C_1 and C_3 , being connected with the iron plate, form a thermo-

electric couple, and if the action of the magnet should make a difference of temperature amounting to 0.01 degree between the junctions (1) and (3), the thermo-electric action of this copper-iron couple would produce as great a deflection of the galvanometer as that given above, which was taken to indicate a change of resistance. Accordingly, it was necessary to consider the possible magnitude of this second longitudinal effect, which will presently be considered.

It is to be noted that Plate 1 shows in a field of 10,700 an *increase* of resistance which we may represent, on an arbitrary scale, by 16, while Plate 2 shows in a field of 5400 a *decrease* of resistance which we may represent on the same scale by 3. Zahn says, "In ferromagnetic metals there is in weak fields an increase of resistance, in stronger fields a decrease. According to the results of Grunmach for iron and cobalt and a preliminary publication of Blake on measurements in nickel, the initial increase is perhaps due to longitudinal components [of magnetism]." When it is remembered that in one of our plates, (1), the resistance is measured at right angles with the fibres or grain of the iron, while in (2) it is measured in the direction of the fibres, it need not, perhaps, be considered strange that the change of resistance caused by magnetic action is opposite in direction in the two cases. In both cases the change is very small.

We now proceed to the other longitudinal effect which is so intimately connected with the one just discussed.

Longitudinal Electromagnetic Temperature-Difference. — With the magnetic field and the *EE* current as in Figure 1, we may look for a difference of temperature, $\Delta\theta'$, established by magnetic action between two points d cm. apart along the plate. We have spent much time in looking for this effect, especially in Plate 1, being led on by accidental phenomena which simulated it rather persistently, though with discrepancies that were suspicious, until a slight change in the disposition of the plate between the poles of the magnet wiped out the apparent temperature change which we had been studying and possibly gave a slight apparent change in the opposite direction.

With Plate 2 we gave only one day, a whole day, to this particular question. The method used was the following: Copper-constantan couple (1), having one junction at point (1) on the plate and the other in a glass tube placed in a water-bath, was put in circuit with one coil of our galvanometer. Couple (3), with one junction at point (3) on the plate and the other in the water-bath, was put in circuit with the other coil of the galvanometer, opposing the action of couple (1) and nearly balancing it. Then the magnetic field was brought into action and the consequent deflections of the galvanometer were observed.

The day's work gave a net deflection of 0.03 cm. as the result of magnetic action, — a quantity smaller than its own probable error. This deflection, if considered reliable, would indicate a temperature-difference of 0.0008°C. , — about a tenth of the temperature-difference that would be needed to account for the apparent change of resistance which has been described above.

The coefficient of the longitudinal effect here looked for but not found may be defined as follows :

$$eL_h = \frac{\Delta\theta'}{d} \div \frac{C_p}{wt} H. \quad (22)$$

Longitudinal Thermomagnetic Potential-Difference. — With the magnetic field and the HH current as in Figure 3, we may take note of the difference of potential, $\Delta P'$, set up by magnetic action between two points d cm. apart along the plate. We shall call this effect positive when the potential-gradient thus established is in the same direction as the temperature-gradient along the plate. As the coefficient of this effect we have

$${}_nL_e = \frac{\Delta P'}{d} \div \frac{d\theta}{dl} H, \quad (23)$$

where $\frac{d\theta}{dl}$ means the temperature-gradient along the plate.

We have not looked for this effect in Plate 2. In Plate 1 we studied it at considerable length, because of a disagreement between our observations and those reported by Zahn on the authority of Houllevigue and of Moreau. He says, "The phenomenon is complicated here [in ferromagnetic metals], for the sign is dependent on the field-strength. With small H the potential-gradient established is in the same direction as the temperature-gradient; it reaches a maximum at ordinary temperature [*bei mittlerer Temperatur*] for a field-strength of about 3000 gauss, then vanishes, in nickel for H about 4500, in iron and steel at about double this field-strength, and takes, for higher fields, the opposite direction. The reversal of sign occurs at a lower field-strength the higher the mean temperature of the plate is."

We used at first the iron wires I_1 and I_3 , connected with points (1) and (3) on the plate. The outer ends of these wires were permanently joined to copper wires, in junctions like the one shown in Figure 10, and from these junctions the copper wires led to the galvanometer. After such a balance of thermo-electric forces in this circuit as to keep the scale of the galvanometer in the field of vision had been effected, the magnetic force was brought into action. An effect was observed ;

but we soon found that the thermo-electric behavior of the iron wires, as well as that of the plate, was affected in such a way by the magnetization that it was impossible to get satisfactory results with this arrangement.

Accordingly we attached two other iron wires, one to each of the copper blocks *B* and *B* of Figure 7, near the ends of the iron plate but well outside the powerful intrapolar region of the magnetic field, within which the iron wires previously used lay. Working with this arrangement we got very considerable effects which showed that transverse magnetization produced in the plate a change of quality or condition such as to make the copper block at the hotter end of the plate electrically positive compared with the copper block at the colder end. This difference of potential, $\Delta P'$, we could and did measure, but there was, and still is, some uncertainty as to the value which should be assigned to d in this case. It should not be the whole length of the plate, for the whole length was not subjected to the full strength, H , of the magnetic field. We have taken d as 80 per cent of the whole length of the plate between the blocks, and have reckoned ${}_hL_e$ accordingly. Each value of this coefficient, as given below, was found from two sets of observations, one made with the heat-current flowing south, the other with it flowing north. As usual, the numbers given in the last three columns are approximate only.

1910.	${}_hL_e$.	Arm θ .	$\frac{d\theta}{dt}$.	H .
June	80×10^{-5}	47°	13.7	12000
14-17	83 "	47°	13.7	8200
	211 "	47°	13.7	2000
28-29	69 "	48°	13	12000
	73 "	48°	13	7900
	187 "	48°	13	1900

The relation of ${}_hL_e$ to H in these two groups of data is shown by means of the two curves of Figure 14. The $\times$ points are for the earlier observations, the $\circ$ points for the later ones. The two curves are very like, and neither suggests a reversal of sign of the effect in question at any stage of magnetization. Why one curve runs notably lower than the other through its whole course we cannot say with full confidence. It is, however, a fact that in the period of the June 14-17 observations the arrangement for getting the sensitiveness of the galvanometer was under suspicion. In the warm, moist atmosphere of a Cambridge summer the problem of insulation in delicate electrical measurements becomes considerable. Leakages occur to a disturbing extent where

during the months of artificial heating of the laboratory they need not be regarded. There was perceptible leakage from the magnet-circuit into the galvanometer-circuit during June. Efforts were made to eliminate errors from such disturbances by combining series of observations taken under a variety of conditions, but we cannot be sure that these efforts were entirely successful. It is unlikely, however, that they affected vitally the results at which we arrived.

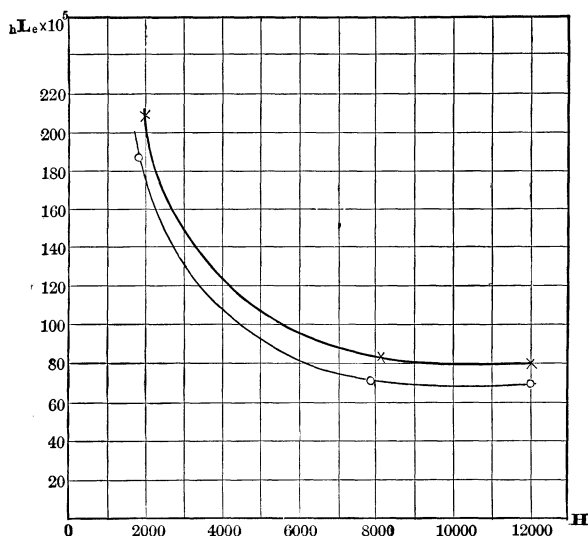


FIGURE 14.

Longitudinal Thermomagnetic Temperature-Difference. — With the magnetic field and the HH current as in Figure 3, we may look for an increase or decrease of the temperature-gradient along the plate as the result of magnetic action. This effect was looked for in Plate 1, but its existence was not proved. The method used was substantially the same as that illustrated in Figure 9 for determining a transverse temperature-difference. That is, circuits (1) and (3), acting thermoelectrically only, were set against each other, each having one coil of the galvanometer. The great initial difference of temperature between the junction (1) and the junction (3) made it necessary to place the outer junctions of the (1) circuit and of the (3) circuit in baths at different temperatures.

We thought at one time, in April, 1910, that we had found a rather

large positive value for the coefficient in question, which may be defined as follows :

$${}_hL_h = \frac{\Delta\theta'}{d} \div \frac{d\theta}{dl}H, \quad (24)$$

$\Delta\theta'$ being the increase of temperature-difference between two points d cm. apart along the plate.

Critical examination of the case, however, showed that in using, as we must with Plate 1, iron wires connected with points (1) and (3), we had brought in a source of error so great as to make the result entirely uncertain.

We hope to try the experiment with Plate 2, using now the constantan-copper junctions (1) and (3).²⁰

SUMMARY.

1. A new and descriptive nomenclature is proposed and used for the various transverse and longitudinal coefficients. (See equations (1), (2), (3), (4), (23), (24), (25), (26).) For example, the Hall-effect coefficient is called ${}_eT_e$, the initial subscript e indicating that the longitudinal current is electrical, the T indicating that the effect observed is transverse, the final subscript e indicating that this transverse effect is electrical. The Ettingshausen coefficient is called ${}_eT_h$, the Nernst coefficient ${}_hT_e$, the Leduc coefficient ${}_hT_h$. Each of the four transverse effects is called $+$ when the current to which it gives rise in the exterior part of a transverse circuit leaves the metal plate in the direction of the "ponderomotive" force acting on this plate or in what would be the direction of this force if longitudinal heat-flow were replaced by longitudinal electric flow. This accords with the usual convention regarding the sign of the Hall effect, but is opposite to the usual convention regarding the sign of the Nernst effect.

2. Measurements of ${}_eT_e$, ${}_eT_h$, ${}_hT_e$, ${}_hT_h$, and of their temperature-coefficients through a small range, have been made in a single plate of soft iron, for which the electrical conductivity, the Thomson-effect coefficient, and the thermo-electric height relative to copper, were already well known through previous study of pieces from the same bar that furnished the plate. (See Table I.)

3. The Moreau formula ${}_eT_e \div \rho = {}_hT_e \div s$, in which ρ is the specific resistance, absolute, of the iron, and s is the number of ergs of heat-energy absorbed by ten coulombs in rising through one centigrade degree in iron (the Thomson effect), is discussed theoretically and is

²⁰ See footnote, p. 664.

tested by means of the data for iron. (See Table II.) The formula is true for one temperature, near 80° C., according to our data, but departs progressively from the truth with descent from this temperature and probably with ascent above it. Below the temperature mentioned the first member of the Moreau equation, as here given, is too small; above this temperature it is, probably, too large. Hence Moreau's theory, which would account for the Hall effect by a deformation of the plate under magnetic action (this deformation producing a real rotation of the equipotential lines of the longitudinal electric current), and would account for the Nernst effect as a like and equal rotation of the Thomson-effect equipotential lines of a longitudinal temperature-flow, appears to be ill-founded and, if taken literally, unsafe, though it is likely to be useful in a suggestive way.

4. Following the suggestion of the Moreau formula, we tested the equation ${}_eT_e \div \rho = {}_hT_h$ for various temperatures, to see whether the isothermal lines of the longitudinal temperature-flow are, in a field of given strength, rotated in the same direction and to the same extent as the equipotential lines of a longitudinal electric flow. (See Table III.) The direction is the same in the two cases, but the equipotential lines are, according to this test, rotated about one and a quarter times as far as the isothermal lines, this ratio remaining nearly constant through the whole range of temperature considered, 20° to 100° . (See Table IV.)

5. "Rotation" cannot account for the Ettingshausen effect, since there is no corresponding longitudinal condition to be rotated, none, that is, which can be regarded as adequate to produce the observed transverse effect. On the other hand, the ratio of the Hall transverse potential-gradient to the Ettingshausen transverse temperature-gradient appears to be almost strictly proportional, through a considerable range of temperature, to the Thomson effect s . Thus we have, approximately, ${}_eT_e \div {}_eT_h = 200 s$, from 20° to 80° or higher. (See Table V.) It should be said, however, that the temperature-coefficient of ${}_eT_h$ cannot be regarded as accurately known. The suggestion is made that the transverse potential-gradient, which is like the Thomson-effect potential-gradient in being *static* (not attended by flow down the gradient), may be the cause of the transverse temperature-gradient, whereas in the Thomson effect the temperature-gradient causes the potential-gradient. It seems likely that the electron theory must here be used, but the attempt is not made in this paper.

6. The Voigt formula ${}_eT_e \div \rho = {}_hT_e \div \Theta'$, in which Θ' , the "thermo-electric height" of lead relative to the metal, takes the place of the Thomson effect s of the Moreau formula, is considered. The iron here

studied has never been tested against lead directly, but from Matthiessen's value of the thermo-electric height of "galvanoplastic" copper relative to lead, and from the known thermo-electric height of this iron relative to copper wire of the present day, values of Θ' have been estimated for various temperatures, from 20° to 100° . With these values of Θ' , and with values of the other factors taken from or reckoned from direct observations on the iron here studied, the Voigt formula has been tested. (See Table VII.) Like the Moreau formula, it seems to be correct at one temperature, between 80° and 90° , and to be untrue at other temperatures. Unlike the Moreau formula, it fails because the left-hand member is too large below this particular temperature. No constant added to the value of Θ' would make the formula hold true with varying temperature. In this connection it is pointed out that, the ordinary thermo-electric diagram being merely a temperature-entropy diagram (with the temperature-axis horizontal and the entropy-axis vertical, unfortunately contrary to the familiar custom of ordinary thermodynamics), thermo-electric heights are merely entropy-differences. Accordingly, Voigt's Θ' , the thermo-electric height of lead relative to any metal, cannot be regarded as a fundamental datum for the metal; for certainly we cannot suppose the entropy of electricity to be zero in lead, as we know there are metals which have a negative thermo-electric height, a negative electric entropy, relative to lead.

7. A thermo-electric diagram is given for copper and iron in which the temperature-axis is made vertical and the entropy-axis horizontal. This diagram shows graphically, by curvature of the iron-line, the law of change of the Thomson-effect coefficient with change of temperature, which law was in an earlier paper expressed algebraically. (See Figure 12.)

8. The longitudinal effect which is shown as a change of resistance of the iron plate by magnetization in the direction of its thickness, the coefficient of which effect is called ${}_eL_e$, was observed in two iron plates, one cut with its width parallel to the fibres, or grain, of the iron, the other with its length parallel to these fibres. In the former plate there was a very slight *increase* of resistance in a field of 10,700 absolute units, in the latter a still slighter *decrease* of resistance in a field of 5400.

9. The longitudinal effect which is shown as a change of longitudinal potential-gradient, due to magnetization in the direction of the thickness of the plate, in a plate along which a heat-current is flowing, was observed in one of the plates mentioned in (8), the one first described there. It was not looked for in the other plate. The coefficient of this effect is called ${}_hL_e$. (See equation (25).) According to Zahn, who quotes Houllevigue and Moreau as authorities, the potential-

gradient established in iron by magnetic action is in the same direction as the temperature-gradient in comparatively weak fields, up to perhaps 9000, but is in the opposite direction in stronger fields. In the iron here studied the potential-gradient produced by the magnetization is in the same direction as the temperature-gradient all the way from a field-strength of 1900, the lowest tried, to 12000, the highest tried, though the coefficient ${}_hL_e$ is much larger for weak fields than for strong fields (see Figure 14).

10. The other two longitudinal effects, the coefficients of which may be called ${}_eL_h$ and ${}_hL_h$, were looked for diligently in one plate, but their existence there was not proven.²¹

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Postscript added March 11, 1911. — The objection which we have made to the use of Θ'_m , the “thermo-electric height” of lead with respect to any metal m , as a fundamental property of the metal m , seems to me so plain that I have been at great pains to make sure that we have not fallen into some gross blunder of interpretation of Voigt’s theory. There can be no question, however, that Moreau takes Voigt’s Θ'_m in this way, and I have seen no protest against his doing so. There is possibly room for doubt as to whether Voigt had *lead* in mind or some other, unnamed, material of zero Thomson effect, with reference to which his Θ'_m is to be reckoned, but this question is of little importance.

Apparently Moreau does not criticise Voigt’s theoretical discussion. He seems merely to point out that it is different from his own, and to test each empirically by means of data which he has taken from various sources. He says, “I consider a circuit [*chaîne*] formed by a metal M and lead, the junctions being at different temperatures. At a point (x, y, z) of M , if X, Y, Z are the components of the thermo-electric electromotive force per unit of length, one has, according to M. Voigt,

$$X = \Theta' \frac{\partial T}{\partial x}, \quad Y = \Theta' \frac{\partial T}{\partial y}, \quad Z = \Theta' \frac{\partial T}{\partial z}, \quad \text{where } \Theta' = \frac{\partial \Theta}{\partial T}. \quad (1)$$

Θ is a function of the absolute temperature T , and is characteristic of the metal M .”

“Let us suppose that a thin slice of the metal M is placed in a mag-

²¹ It now appears that magnetization parallel to the thickness decreases notably the longitudinal temperature-gradient in Plate 2. March 11, 1911.

netic field H , perpendicular to the lines of force, the plane XY coinciding with the plane of the slice, which will be traversed by a current of heat going in the direction OY . Equations (1) give

$$X = 0, \quad Y = \Theta'_m \frac{\partial T}{\partial Y}, \quad Z = 0.$$

“M. Voigt supposes that the force Y turns under the action of the field as the electromotive force of the primary current in the Hall phenomenon turns. One obtains, then, along the axis OX , a transverse electromotive force X , which, referred to the unit of length, is

$$X_1 = -\frac{C}{\rho} \Theta'_m \frac{\partial T}{\partial y} H$$

(where C is the Hall coefficient, ρ the resistivity of the metal), or, according to equation (3) [$\Phi = \Theta'_m$],

$$X_1 = -\frac{C}{\rho} \Phi \frac{\partial T}{\partial y} H. \quad (4)$$

“This formula (4) gives the thermo-electric effect according to M. Voigt.”

We thus get as the expression, according to Voigt, for K , the Nernst coefficient,

$$K^{22} = X_1 \div \frac{\partial T}{\partial y} H = -\frac{C}{\rho} \Phi,$$

and this is the expression which Moreau tests by means of data which he gives. For example, he gives the value of Φ as -1619 for iron and -152 for copper, evidently taking Φ as the thermo-electric height of *lead with respect to the other metals*.

As to his own formula for K , Moreau says, contrasting his point of view with that of Voigt, “By assuming that, *only*, the thermo-electric electromotive forces relative to the Thomson effect turn under the action of the field, I have obtained the formula

²² Moreau does not expressly say that $K = -\frac{C}{\rho} \Phi$ according to Voigt. He does, however, put the values of $-\frac{C}{\rho} \Phi$ alongside his own values of K for comparison. Moreau may have been in doubt as to the sign of the Voigt formula.

$$X_2 = KH \frac{\partial T}{\partial y}, \quad \text{or} \quad K = \frac{\sigma C}{\rho},$$

σ being the specific heat of electricity."

Moreau, then, seems to admit that Voigt's Y is a real thermo-electric force,

$$\frac{\partial \Theta_m}{\partial T} \frac{\partial T}{\partial y}, \quad \text{or} \quad \frac{\partial \Theta}{\partial y},$$

directed along the Y -axis at any point in the metal M , though not the force which he considers significant in the Nernst effect. But it seems to me that this supposed total electromotive force at any point in the metal M is a fiction. The only electromotive forces which we have reason to suppose existing in an unequally heated piece of metal *in open circuit*, as a metal is when tested for the Nernst effect, are those found in the Thomson effect. When we have two unequally heated metals united in a thermo-electric circuit, the total electromotive force at any point in either metal is likely to be something different from that which the Thomson effect alone at that point would account for, but we have no sufficient reason for supposing it to be actually the $\frac{\partial \Theta_m}{\partial T} \frac{\partial T}{\partial y}$ mentioned above. If, for example, we are considering iron and if, for simplicity, we assume that $\partial T / \partial y = 1$, the thermo-electric e. m. f. in the iron at temperature T is, according to Voigt, represented by the length of the line TI , taken as a negative quantity, in Figure 11 of our paper. Now it is true that, if we make this assumption and then integrate around the whole circuit, which we will suppose to be of copper and iron, we shall get the area $C_0CH_0C_0$ as the total e. m. f. of the circuit, which result will be correct. But we should arrive at precisely the same correct integral result if we placed the T_0T line of Figure 11 indefinitely far to the left or to the right of its present position, which would have the effect of increasing or decreasing indefinitely the value of the expression $\partial \Theta_m / \partial T$, the supposed force at any individual point in either metal. I can see, then, no objective reality in Voigt's supposed force Y .

Moreau was, I think, the first to put Voigt's formula for the value of the Nernst coefficient to the numerical test. He gives,²³ the three bottom lines being from his own observations,

²³ C. R., **130**, 564 (1900).-

	C_{20} . (Nernst.)	Φ_{20} .	$-\frac{C}{\rho}\Phi_{20}$.	$K = \frac{\sigma C}{\rho}$.	K (obs.). (Nernst.)	$[\sigma_{20}^{24}]$
Bismuth	-10.1	+8800 or +6400	+0.337 or +0.246	+0.149	+0.196	-1875
Antimony	+0.192	-2240	+0.0097	+0.0090	+0.0094	+2074
Nickel	-0.024	+2280	+0.0039	+0.0026	+0.0073	-1482
Cobalt	+0.0046	+2200	-0.00094	-0.00175	+0.00154	-4073
Iron	+0.0113	-1619	+0.0018	-0.00156	-0.00156	-1412
Steel	+0.0175	-1731	+0.00114	-0.00062	-0.00060	- 952
Copper	-0.00052	-152	-0.000047	-0.000084	-0.000073	+ 275
Zinc	-0.00041	-279	-0.000018	-0.000046	-0.000054	+ 697
	C_0 .	Φ_0 .	$-\frac{C_0}{\rho_0}\Phi_0$.		K_0 (obs.).	
Soft iron	+0.00837	-1062	+0.000636		-0.000646	
Soft steel	+0.00662	-1351	+0.000814		-0.000596	
Cobalt	+0.00399	+1318	-0.000512		-0.00146	

It will be seen that K , which = our ${}_nT_e$ and Zahn's $-Q$, as calculated from the formula of Moreau and given in the fourth column agrees in sign with K as observed by Nernst in every case except that of cobalt, and that in this case direct observation by Moreau himself gives the same sign which his formula gives. In other words, according to Moreau his own formula gives the right sign for K in every case. Zahn, however, agrees with Nernst in his observation of the sign of the Nernst effect in cobalt.

Moreau finds the Voigt formula, or at least the expression $-\frac{C\Phi}{\rho}$, to give the same signs as his own formula in all the cases examined except those of iron and steel, but I am inclined to the opinion that he has made a mistake of sign in the use of the Voigt formula. It seems to me that the Voigt formula gives the right sign for K in iron and

²⁴ C. R., 130, 124 (1900).

steel and the wrong sign in the other cases. I have not found any perfectly explicit statement by Voigt himself of the relation between the Nernst coefficient and the Hall coefficient, but I reason as follows: If we consider iron and take a case in which $\partial T/\partial y$ is positive, we have, as we saw a little distance back, Y as a negative quantity; that is, according to Voigt, there is a thermo-electric e. m. f. acting in the direction of decreasing y . But with our iron in open circuit, as it would be for the Nernst-effect observations, we have no flow of electricity along the iron, the natural thermo-electric e. m. f. being balanced, or, as Voigt says, *compensated*, by a static charge having a negative potential-gradient in the positive direction of y . If, now, we think of the magnetic field as rotating the equipotential lines of this potential-gradient in the direction of the Amperian current, as the equipotential lines of the primary electric current are rotated in the Hall effect in iron, we get what in our paper has been called, and what Moreau would call, a *negative* Nernst effect.

According to this argument, which deals with a puzzling question and may be incorrect in its conclusion, all the signs in the third column of Moreau's table, as given above, should be changed, and in every case for which the specific heat of electricity within a metal and the thermo-electric height of the metal with respect to lead have the same sign the Moreau formula and the Voigt formula will predict opposite signs for the Nernst effect.

The Moreau formula seems to me profoundly suggestive, though not strictly correct. But the fact that the Voigt formula, which I believe to be fundamentally wrong in one of its factors, gives results numerically so like those of Moreau, and so like the values given by direct observation, is enough to warn us to be cautious in examining every theory and testing every formula relating to the matters here dealt with.

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